Perfect information games on random trees

Perfect information games are widely used in Economics to study peace talks, legislative bargaining, individual consumption and savings decisions, environmental policies, electoral competition, and dynamic perfect competition. Another prominent area of application of perfect information games is Computer Science.

Almost invariably, researchers study games played over very nicely structured, well-behaved game trees. Such carefully designed games, however, represent only a small fraction of all possible games. In contrast to the traditional approach, we take a probabilistic viewpoint on perfect information games. We generate the game tree randomly, and analyze the behavior of a "typical" game. This allows us to address such questions as: what is the probability for a player to win? What is the distribution of the payoffs? What is the maximal payoff a player could obtain with probability 1? The project aims to develop a probabilistic approach to games, and to make it amenable to applications in economics and in computer science.

Another ambition of this project is to build a bridge between game theorists and computer scientists working on perfect information games. The work produced in recent years by the two groups of researchers converged remarkably, displaying common interests, methods, and applications. And yet the two groups remain surprising isolated from one another. Oftentimes, game theorists would remain oblivious of the contributions of computer scientists, and vice versa. This project, lying at the interface of game theory and computer science, is an ideal platform for exchange of ideas and cooperation.

Keywords: perfect information game, random graph

Research idea

- *Aims*

We aim to develop a new, probabilistic, approach to perfect information games. Mostly, researchers study games played over very nicely structured, well-behaved game trees. In contrast, we would like to understand the behaviour of a "typical", randomly chosen game.

Consider the following example. Alice and Bob move through the nodes of a tree. The game is adversarial. Alice's goal is to avoid terminal nodes, that is, the nodes having no outgoing branches. Bob's goal is to reach such a node. We generate the game tree as follows: starting with a binary tree, we declare each node (independently) to be terminal with probability 1/4. We then assign each of the remaining nodes (independently) to either Alice (with probability 1/2), or to Bob (with probability 1/2). Both players see the game tree before they start playing the game. Figure 1 depicts one particular realization of the game. Under this realization Bob wins, his winning strategy shown in bold.

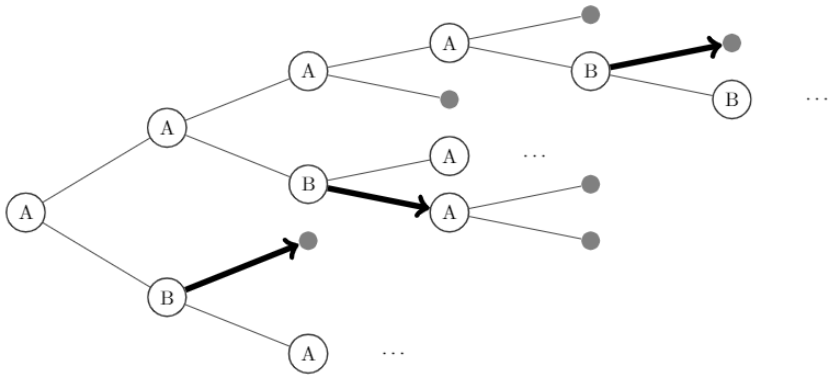

Figure 1: A realization of the random game. The dots indicate that the game tree continues. Bob's strategy is shown in bold.

Does Alice have a chance to win? No: In our example, Bob wins with probability 1. Moreover, we can estimate the time needed for Bob to reach a terminal node: he can do so by period 12 with probability of at least 0.95. If we assign Alice to a node with the probability of 3/4, she wins with probability 1/3. The game displays an interesting phase transition: Alice has a positive chance of winning the game if and only if she is assigned to a node with a probability larger than 2/3.

One promising application of the new approach is to graph games, a class of games played on graphs, the importance of which stems from its use in computer science. To make this and other applications feasible, we plan to: (a) consider games with classical computer science objectives, such as reachability and safety; consider qualitative as well as quantitative objectives (i.e., payoff functions); (b) go beyond zero-sum games and treat *n*-player games; (c) go beyond random trees, and consider games played on other types of random graphs.

Innovative aspects

The random game tree is the main novel feature of our approach. This is in contrast to much of applied and theoretical work where a game tree would be carefully designed to serve the researcher's needs. This traditional approach tends to focus on very particular games that are hardly representative of a "typical" game. The behaviour of an "average" game is at the heart of our approach.

Urgency

Particular examples of perfect information games on random trees have recently appeared in the literature (see references below) in connection with problems of percolation theory. The idea has not yet penetrated other fields. There is thus a narrow window of opportunity for making a significant contribution to game theory and computer science. The method is potentially very versatile and applicable beyond the specific context where it emerged. We aspire to make this project a stepping-stone to these developments.

Methodology

Provide a concise and general description of your methodological approach.

The methodology draws from two areas of research: perfect information games and random graphs. We combine the insights from both fields to study perfect information games on randomly generated trees. Thus, the game tree is generated randomly using one of the methods from probability theory, for example by a percolation of a given graph (like in the example above), or a branching process. The players are randomly assigned to the nodes. The players observe the realized tree, and play the game. Finally, we analyze the resulting solution from a statistical perspective.